Calculator Free
Integration, Fundamental Theorem of Calculus, Area

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 2, 2 =8 marks] **CF**

(a) Calculate $\int \cos\left(\frac{t}{3}\right) dt$

(b) Use your answer to part (a) to evaluate $\int_{\pi}^{2x+1} \cos\left(\frac{t}{3}\right) dt$, in terms of x

(c) Use your answer to part (b) to evaluate $\frac{d}{dx}\left(\int_{\pi}^{2x+1} \cos\left(\frac{t}{3}\right) dt\right)$

(d) Hence evaluate $\frac{d}{dx}\left(\int_{\pi}^{f(x)} \cos\left(\frac{t}{3}\right) dt\right)$

Question Two: [2, 2, 2 = 6 marks] CF

Determine each of the following:

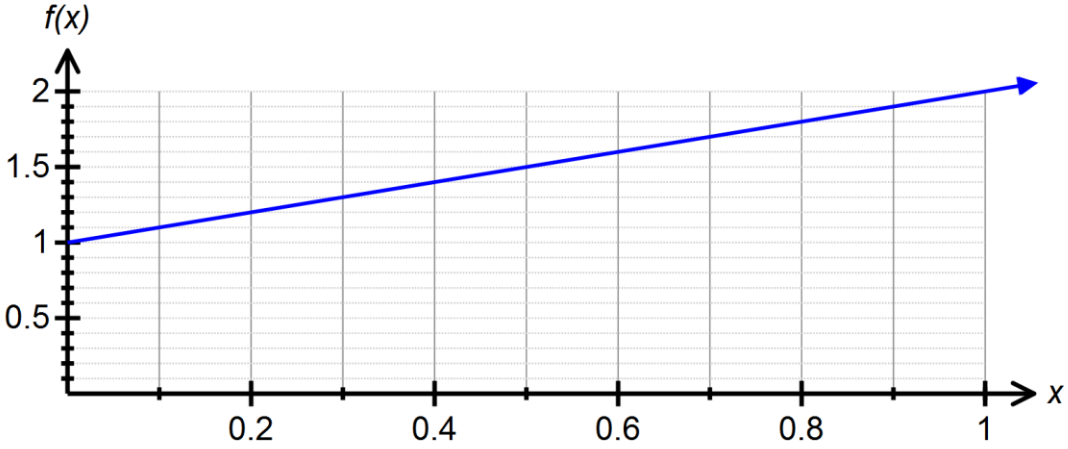
(a) $\int_{-1}^{1} 2x^3 \, dx$

(b) $\int_{-1}^{0} e^x \, dx - \int_{1}^{0} e^x \, dx$

(c) $\frac{d}{dx}\left(\int_{-3}^{x^2} \frac{\sqrt{2t-3}}{t+1} \, dt \right)$

Question Three: [2, 3, 2 = 7 marks] **CF**

Consider the function $f(x)$ drawn below over the domain $0 \le x \le 1$



(a) Draw rectangles on your graph that can be used to underestimate the area under $f(x)$ over the domain $0 \le x \le 1$, where $\delta x = 0.2$.

(b) Show that $\sum_5 f(x_5)\delta x_5 = \frac{7}{5}\ units^2$

(c) Use the graph of $f(x)$ above to calculate $\int_0^1 f(x)\ dx$

Question Four: [4, 5 = 9 marks] CF

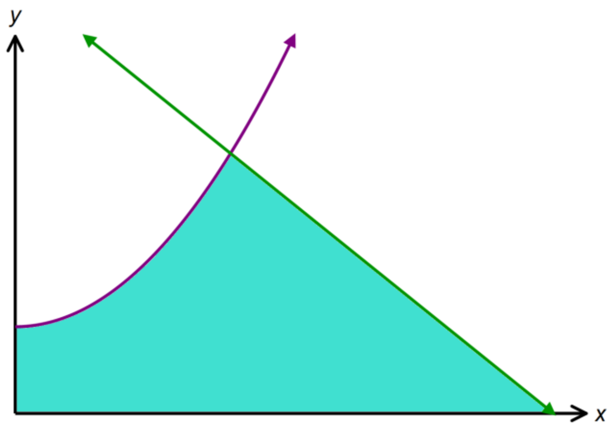
Consider the function $f(x) = x^3 + 2x^2 - x - 2$

(a) Determine the roots of the function.

(b) Hence determine the area bounded by the curve and the x – axis.

Question Five: [1, 2, 4 = 7 marks] CF

The functions $f(x) = x^2 + 2$ and $h(x) = -2x + 10$ are drawn below.



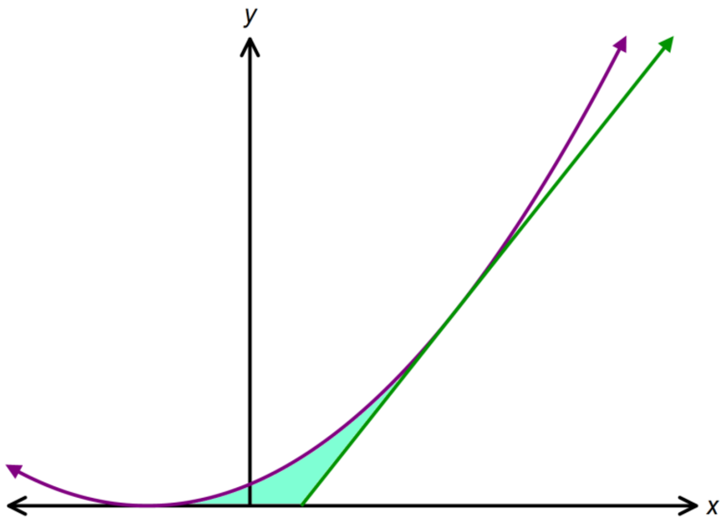
(a) Solve $h(x) = 0$

(b) Solve $f(x) = h(x)$

(c) Hence find the area shaded on the graph above.

Question Six: **[3, 5 = 8 marks]** **CF**

The curve $y=(x+1)^2$ and the tangent line at $x=2$ are graphed below.



(a) Determine the equation of the tangent to $y=(x+1)^2$ drawn above.

(b) Hence find the area shaded on the graph above.

SOLUTIONS
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Question One: [2, 2, 2, 2 =8 marks] **CF**

(a) Calculate $\int \cos\left(\frac{t}{3}\right)dt$

$$= 3\sin\frac{t}{3} + c$$

(b) Use your answer to part (a) to evaluate $\int_{\pi}^{2x+1} \cos\left(\frac{t}{3}\right)dt$, in terms of x

$$= \left[3\sin\frac{t}{3} + c\right]_{\pi}^{2x+1}$$

$$= \left(3\sin\frac{2x+1}{3} + c\right) - \left(3\sin\frac{\pi}{3} + c\right)$$

$$= 3\sin\frac{2x+1}{3} - \frac{3\sqrt{3}}{2}$$

(c) Use your answer to part (b) to evaluate $\frac{d}{dx}\left(\int_{\pi}^{2x+1} \cos\left(\frac{t}{3}\right)dt\right)$

$$\frac{d}{dx}\left(3\sin\frac{2x+1}{3} - \frac{3\sqrt{3}}{2}\right)$$

$$= 3\cos\frac{2x+1}{3} \times 2$$

$$= 6\cos\frac{2x+1}{3}$$

(d) Hence evaluate $\frac{d}{dx}\left(\int_{\pi}^{f(x)} \cos\left(\frac{t}{3}\right)dt\right)$

$$= \cos\left(\frac{f(x)}{3}\right) \times f'(x)$$

Question Two: [2, 2, 2 = 6 marks] CF

Determine each of the following:

(a) $\int_{-1}^{1} 2x^3 \, dx$

$$= \left[\frac{2x^4}{4}\right]_{-1}^{1}$$

$$= \frac{1}{2} - \frac{1}{2}$$

$$= 0$$

(b) $\int_{-1}^{0} e^x \, dx - \int_{1}^{0} e^x \, dx$

$$= \int_{-1}^{0} e^x \, dx + \int_{0}^{1} e^x \, dx$$

$$= \int_{-1}^{1} e^x \, dx$$

$$= \left[e^x\right]_{-1}^{1}$$

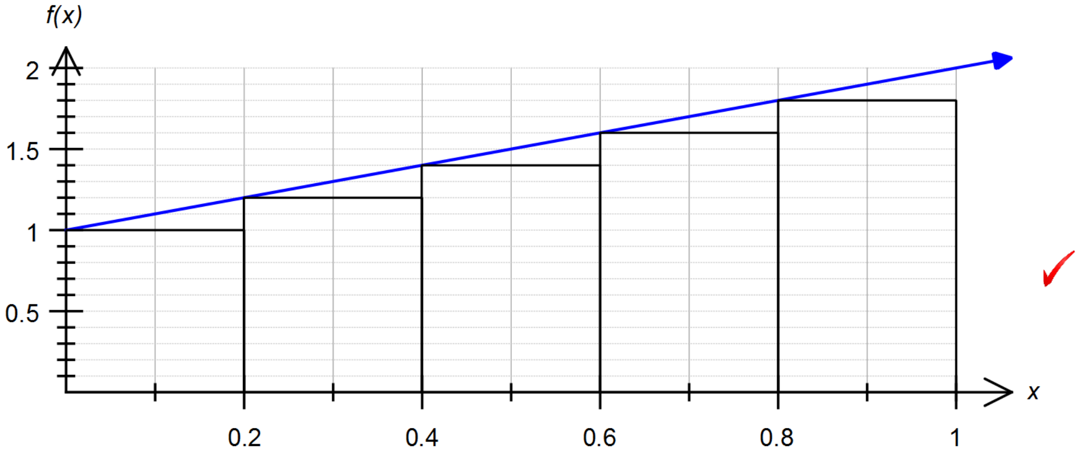
$$= e^1 - e^{-1}$$

(c) $\frac{d}{dx}\left(\int_{-3}^{x^2} \frac{\sqrt{2t-3}}{t+1} \, dt\right)$

$$= \frac{\sqrt{2x^2-3}}{x^2+1} \times 2x$$

Question Three: [2, 3, 2 = 7 marks] CF

Consider the function $f(x)$ drawn below over the domain $0 \le x \le 1$



(a) Draw rectangles on your graph that can be used to underestimate the area under $f(x)$ over the domain $0 \le x \le 1$, where $\delta x = 0.2$.

(b) Show that $\sum_5 f(x_5)\delta x_5 = \frac{7}{5}\ units^2$

$\sum_5 f(x_5)\partial x_5 = 0.2\times1 + 0.2\times1.2 + 0.2\times1.4 + 0.2\times1.6 + 0.2\times1.6$ ✓

$= 0.2(1+1.2+1.4+1.6+1.8)$ ✓

$= \frac{1}{5}\times 7$ ✓

$= \frac{7}{5}\ units^2$

(c) Use the graph of $f(x)$ above to calculate $\int_0^1 f(x)\ dx$

$= \frac{1(1+2)}{2} = \frac{3}{2}$ ✓
✓

Question Four: [4, 5 = 9 marks] CF

Consider the function $f(x)=x^3+2x^2-x-2$

(a) Determine the roots of the function.

$x=1\, is\, a\;\; factor$ ✓

$$\begin{array}{r} x^2+3x+2 \\ x-1\overline{)x^3+2x^2-x-2} \\ \underline{x^3-x^2} \\ 3x^2-x \\ \underline{3x^2-3x} \\ 2x-2 \\ \underline{2x-2} \\ 0 \end{array}$$ ✓

$f(x)=(x-1)(x^2+3x+2)$

$f(x)=(x-1)(x+2)(x+1)$ ✓

$roots=(1,0)\;(-2,0)\;(-1,0)$ ✓

(b) Hence determine the area bounded by the curve and the x – axis.

$$=\int_{-2}^{-1} f(x)\;dx+\left|\int_{-1}^{1} f(x)\,dx\right|$$ ✓

$$=\left[\frac{x^4}{4}+\frac{2x^3}{3}-\frac{x^2}{2}-2x\right]_{-2}^{-1}+\left|\left[\frac{x^4}{4}+\frac{2x^3}{3}-\frac{x^2}{2}-2x\right]_{-1}^{1}\right|$$ ✓

$$=\left(\frac{1}{4}-\frac{2}{3}-\frac{1}{2}+2\right)-\left(4-\frac{16}{3}-2+4\right)+\left|\left(\frac{1}{4}+\frac{2}{3}-\frac{1}{2}-2\right)-\left(\frac{1}{4}-\frac{2}{3}-\frac{1}{2}+2\right)\right|$$ ✓

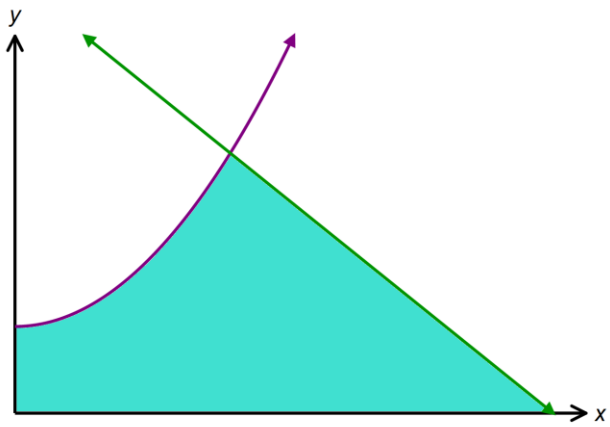
$$=\frac{-1}{4}-4+\frac{14}{3}+\left|\frac{4}{3}-4\right|$$

$$=\frac{5}{12}+2\frac{2}{3}$$ ✓

$$=3\frac{1}{12}\,units^2$$ ✓

Question Five: [1, 2, 4 = 7 marks] CF

The functions $f(x)=x^2+2$ and $h(x)=-2x+10$ are drawn below.



(a) Solve $h(x)=0$

$-2x+10=0$

$-2x=-10$

$x=5$ ✓

(b) Solve $f(x)=h(x)$

$x^2+2=-2x+10$

$x^2+2x-8=0$ ✓

$(x+4)(x-2)=0$

$x=-4,\ x=2$ ✓

(c) Hence find the area shaded on the graph above.

$$Area=\int_0^2 x^2+2\ dx+\int_2^5 -2x+10\ dx$$ ✓

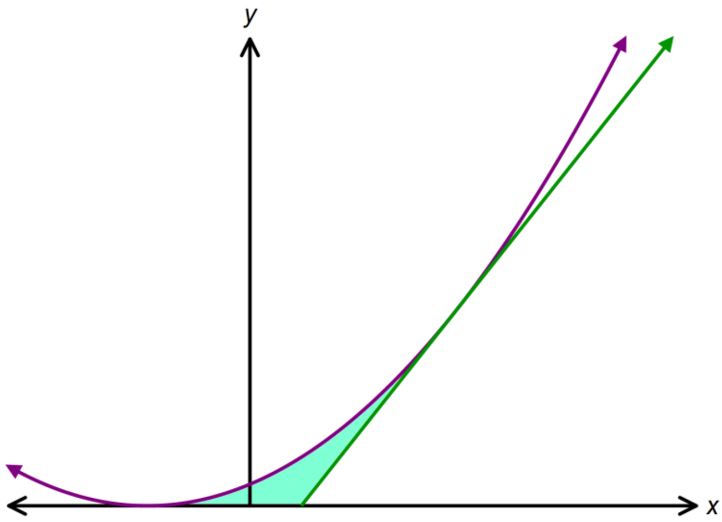
$$=\left[\frac{x^3}{3}+2x\right]_0^2+\left[-x^2+10x\right]_2^5$$ ✓

$$=\left(\frac{8}{3}+4\right)-(0+0)+(-25+50)-(-4+20)$$ ✓

$$=15\frac{2}{3}\ units^2$$ ✓

Question Six: [3, 5 = 8 marks] CF

The curve $y=(x+1)^2$ and the tangent line at $x=2$ are graphed below.



(a) Determine the equation of the tangent to $y=(x+1)^2$ drawn above.

$$\frac{dy}{dx}=2(x+1)$$
$$x=2 \quad \frac{dy}{dx}=2(2+1)=6$$
$$x=2 \quad y=(2+1)^2=9$$
$$y=6x+c$$
$$9=6\times 2+c$$
$$c=-3$$
$$\therefore y=6x-3$$

(b) Hence find the area shaded on the graph above.

$$Area=\int_{-1}^{2}(x+1)^2\ dx-\int_{0.5}^{2}6x-3\ dx$$
$$=\left[\frac{(x+1)^3}{3}\right]_{-1}^{2}-\left[3x^2-3x\right]_{0.5}^{2}$$
$$=(9+0)-\left(6+\frac{3}{4}\right)$$
$$=2\frac{1}{4}\ units^2$$